

07. ઘાત અને ઘાતમૂળ

મિત્રો, આ પ્રકરણને સમજવા માટે નીચેના કેટલાક નિયમો પહેલા સમજીશું.

01. $[x^0 = 1]$ કોઈપણ સંખ્યાની શૂન્ય ઘાતની કિંમત 1 થાય.

ઉદા.: $5^0 = 1, \quad 4^0 = 1$
 $-1^0 = 1, \quad -254^0 = 1$

02. $[1^x = 1]$ 1 ની કોઈપણ ઘાત કરવાથી તેનું મૂલ્ય 1 થાય.

ઉદા.: $1^5 = 1, \quad 1^{251} = 1$
 $1^{-100} = 1$

03. $[x^1 = x]$ કોઈપણ સંખ્યાની 1 ઘાત કરવાથી તેનું મૂલ્ય તે સંખ્યા જ રહે છે.

ઉદા.: $5^1 = 5, \quad 6^1 = 6$
 $-2^1 = -2$

04. $[x^a \times x^b \times x^c = x^{a+b+c}]$ જે ઘાતવાળી સંખ્યાના આધાર સરખા હોય તેનું સાદુરૂપમાં ઘાતોનો સરવાળો થાય.

ઉદા.: $5^2 \times 5^7 \times 5^5 = 5^{2+7+5} = 5^{14}$

05. $\left[\frac{x^a}{x^b} = x^{a-b}\right]$ ઘાતોની બાદબાકી થાય.

ઉદા.: $\frac{5^7}{5^3} = 5^{7-3} = 5^4$

06. $[x^a y^b z^c]^d = x^{ad} y^{bd} z^{cd}]$ કૌંસના ઉપરની ઘાત કૌંસની અંદરની તમામ સંખ્યાની ઘાત સાથે ગુણાય.

ઉદા.: $(5^2 \times 4^3 \times 7^2)^4 = 5^{(2)(4)} \times 4^{(3)(4)} \times 7^{(2)(4)}$
 $= 5^8 \times 4^{12} \times 7^8$

07. $[x^a]^b = x^{ab}$ ઘાતની ઘાતનો ગુણાકાર થાય.

ઉદા.: $(5^2)^3 = 5^{2 \times 3} = 5^6$

08. $[x^{-y} = \left(\frac{1}{x}\right)^y]$ કોઈપણ સંખ્યાની ઋણઘાત હોય તો આધારનો વ્યસ્ત કરવાથી તે ઘાત ધન થાય.

ઉદા.: $5^{-2} = \left(\frac{1}{5}\right)^2$

■ અન્ય ઉદાહરણો :

01. નીચેમાંથી કઈ કિંમતો સમાન છે ?

(a) 1^4 (b) 4^0 (c) 0^4 (d) 4^1

ગણતરી : $1^4 = 1, \quad 4^0 = 1, \quad 0^4 = 0, \quad 4^1 = 4,$

જવાબ : a અને b ની કિંમત સમાન છે.

02. નીચેમાંથી કોનું મૂલ્ય સૌથી વધારે છે ?

(a) $\left[(4^0)^{-\frac{1}{2}}\right]^2$ (b) $\left[(2^{-1})^0\right]^2$

(c) $\left[(2^{-2})^{-1}\right]^2$ (d) $\left[(2^{-1})^2\right]^2$

ગણતરી: (a) $\left[(4^0)^{-\frac{1}{2}}\right]^2 = 4^{0 \times \frac{1}{2} \times 2} = 4^0 = 1$

(b) $\left[(2^{-1})^0\right]^2 = 2^{-1 \times 0 \times 2} = 2^0 = 1$

(c) $\left[(2^{-2})^{-1}\right]^2 = 2^{(-2) \times (-1) \times 2} = 2^4 = 16$

(d) $\left[(2^{-1})^2\right]^2 = 2^{-1 \times 2 \times 2} = 2^{-4} = \frac{1}{16}$

જવાબ: સૌથી મોટી સંખ્યા વિકલ્પ C છે.

03. જો તો $m^n = 121$ તો $(m-1)^{n+1} =$ _____

$\therefore m^n = 11^2$

$\therefore m = 11$ તથા $n = 2$

$(m-1)^{n+1}$

$\therefore (11-1)^{2+1}$

$\therefore (10)^3 = 1000$

04. $a^{15} \times a^{-8} \times a^{-7} =$ _____

$\therefore a^{15+(-8)+(-7)} = a^{15-15} = a^0 = 1$

05. $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$ નું મૂલ્ય _____ થાય.

$= \left[5(2+3)^3\right]^{\frac{1}{4}} = \left[5(5)^3\right]^{\frac{1}{4}}$

$= \left[5^4\right]^{\frac{1}{4}} = 5^{4 \times \frac{1}{4}} = 5^1 = 5$

06. કિંમત શોધો.

(અ) $(27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{3 \times \frac{2}{3}} = 3^2 = 9$

(બ) $\left(\frac{8}{125}\right)^{-\frac{4}{3}} = \left(\frac{125}{8}\right)^{\frac{4}{3}} = \frac{5^{3 \times \frac{4}{3}}}{2^{3 \times \frac{4}{3}}} = \frac{5^4}{2^4} = \frac{625}{16}$

$$(g) (0.00032)^{\frac{3}{5}} = (0.2^5)^{\frac{3}{5}} = (0.2)^{5 \times \frac{3}{5}} = 0.008$$

$$(h) (256)^{0.16} \times (16)^{0.18} \\ = (2^8)^{0.16} \times (2^4)^{0.18} \\ = 2^{1.28} \times 2^{0.72} = 2^{2.0} = 4$$

$$(i) \frac{9^4 \times 3^5 \times 27^3}{3 \times 81^4} = \frac{(3^2)^4 \times (3)^5 \times (3^3)^3}{3 \times (3^4)^4} \\ = \frac{3^8 \times 3^5 \times 3^9}{3 \times 3^{16}} \\ = \frac{3^{22}}{3^{17}} = 3^5 = 243$$

પ્રેક્ટીસ માટેના દાખલાઓ :

[01] નીચેનાની કિંમત શોધો.

$$01. (256)^{5/4} \\ = (4^4)^{5/4} = (4)^5 \\ = 4 \times 4 \times 4 \times 4 \times 4 = \boxed{1024}$$

$$02. (\sqrt{8})^{1/3} \\ = \sqrt{8^{1/3}} \\ = \sqrt{2}$$

$$03. \left(\frac{32}{243}\right)^{-4/5} \\ = \left(\frac{243}{32}\right)^{4/5} \\ = \left(\frac{3^5}{2^5}\right)^{4/5} \\ = \frac{3^4}{2^4} = \boxed{\frac{81}{16}}$$

$$04. \left(-\frac{1}{216}\right)^{-2/3} \\ = (-216)^{2/3} \\ = (-6^3)^{2/3}$$

$$= (-6)^2 \\ = \boxed{36}$$

$$05. 5^{1/4} \times (125)^{0.25} \\ = 5^{1/4} \times (5^3)^{25/100} \\ = 5^{1/4} \times (5^3)^{1/4} \\ = 5^{1/4} \times 5^{3/4} \\ = 5^{1/4+3/4} = 5^{\frac{1+3}{4}} \\ = 5^{4/4} = \boxed{5}$$

$$06. (216)^{2/3} + (256)^{3/4} + (32)^{1/5} \\ = (6^3)^{2/3} + (4^4)^{3/4} + (2^5)^{1/5} \\ = 6^2 + 4^3 + 2 \\ = 36 + 64 + 2 \\ = \boxed{102}$$

$$07. [(10)^{150} \div (10)^{146}] \\ = 10^{150-146} \\ = 10^4 \\ = \boxed{10000}$$

$$08. (2.4 \times 10^3) \div (8 \times 10^{-2}) \\ = \frac{2.4 \times 10^3}{8 \times 10^{-2}} \\ = \frac{24}{10 \times 8} \times 10^5 \\ = 3 \times 10^4 \\ = \boxed{30000}$$

$$09. \left(\frac{1}{216}\right)^{-2/3} \div \left(\frac{1}{27}\right)^{-4/3} \\ = (216)^{2/3} \div (27)^{4/3} \\ = \frac{(6^3)^{2/3}}{(3^3)^{4/3}} \\ = \frac{6^2}{3^4} = \frac{36}{81} \\ = \boxed{\frac{4}{9}}$$

$$10. \quad \frac{(243)^{0.13} \times (243)^{0.07}}{(7)^{0.25} \times (49)^{0.075} \times (343)^{0.2}}$$

$$= \frac{(3^5)^{0.13} \times (3^5)^{0.07}}{(7)^{0.25} \times (7^2)^{0.075} \times (7^3)^{0.2}}$$

$$= \frac{3^{0.65} \times 3^{0.35}}{7^{0.25} \times 7^{0.15} \times 7^{0.6}}$$

$$= \frac{3^{0.65+0.35}}{7^{0.25+0.15+0.6}}$$

$$= \frac{3^{1.00}}{7^{1.00}} = \boxed{\frac{3}{7}}$$

$$11. \quad (1000)^7 \div (10)^{18}$$

$$= (10^3)^7 \div 10^{18}$$

$$= \frac{10^{21}}{10^{18}}$$

$$= 10^3$$

$$= \boxed{1000}$$

$$12. \quad (256)^{0.16} \times (256)^{0.09}$$

$$= (256)^{0.16+0.09}$$

$$= (256)^{0.25}$$

$$= (4^4)^{\frac{25}{100}}$$

$$= (4^4)^{\frac{1}{4}}$$

$$= \boxed{4}$$

$$13. \quad (0.04)^{-1.5}$$

$$= \left(\frac{1}{0.04} \right)^{-1.5} = \left(\frac{100}{4} \right)^{\frac{15}{10}}$$

$$= (25)^{3/2}$$

$$= (5^2)^{3/2}$$

$$= 5^3$$

$$= \boxed{125}$$

$$14. \quad (17)^{3.5} \times (17)^{(?)} = (17)^8$$

$$\therefore (17)^? = \frac{17^8}{17^{3.5}}$$

$$= 17^{8-3.5}$$

$$17^{(?)} = 17^{4.5}$$

$$\boxed{(?)} = 4.5$$

$$15. \quad (64)^{-1/2} - (-32)^{-4/5}$$

$$= \left(\frac{1}{64} \right)^{1/2} - \left(-\frac{1}{32} \right)^{4/5}$$

$$= \left(\frac{1}{8^2} \right)^{1/2} - \left(-\frac{1}{2^5} \right)^{4/5}$$

$$= \left(\frac{1}{8^2} \right)^{1/2} - \left(\frac{1}{2^5} \right)^{4/5}$$

$$= \frac{1}{8} - \frac{1}{16}$$

$$= \frac{2-1}{16} = \boxed{\frac{1}{16}}$$

$$16. \quad 49 \times 49 \times 49 \times 49 = 7^{(?)}$$

$$\therefore 7^2 \times 7^2 \times 7^2 \times 7^2 = 7^{(?)}$$

$$\therefore 7^{2+2+2+2} = 7^{(?)}$$

$$\therefore 7^8 = 7^{(?)}$$

$$\therefore \boxed{(?)} = 8$$

$$17. \quad (18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^{(?)}$$

$$(18 \div 27 \times 6)^{3.5} = 2^?$$

$$\left(\frac{18 \times 6}{27} \right)^{3.5} = 2^?$$

$$(4)^{3.5} = 2^?$$

$$(2^2)^{3.5} = 2^?$$

$$2^7 = 2^{(?)}$$

$$\boxed{(?)} = 7$$

$$18. \quad (25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^{(?)}$$

$$(5^2)^{7.5} \times (5)^{2.5} \times (5^3)^{1.5} = 5^{(?)}$$

$$\therefore (5)^{15} \times (5)^{2.5} \div 5^{4.5} = 5^{(?)}$$

$$\therefore 5^{15+2.5-4.5} = 5^{(?)}$$

$$\therefore 5^{13} = 5^{?}$$

$$\boxed{13 = (?)}$$

19. જો $\sqrt{2^n} = 64$ તો $n = (?)$

$$\sqrt{2^n} = 64 \text{ બંને બાજુ વર્ગ કરતાં,}$$

$$2^n = (64)^2 = (2^6)^2 = 2^{12}$$

$$\therefore n = 12$$

20. $5^x = 3125$ હોય તો $x-3$ ની કિંમત _____ થાય.

$$5^x = 3125$$

$$\therefore 5^x = 5^5$$

$$\therefore x = 5 \therefore x-3 = 2$$

21. $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$ હોય તો $n = (?)$

$$\therefore \frac{9^n \times 3^5 \times (3^3)^3}{3 \times (3^4)^4} = 3^3$$

$$\therefore \frac{3^{2n} \times 3^5 \times 3^9}{3 \times 3^{16}} = 3^3$$

$$\therefore 3^{2n} \times 3^{14-17} = 3^3$$

$$\therefore 3^{2n} = 3^3 \div 3^{-3}$$

$$3^{2n} = 3^6 \therefore 2n = 6 \therefore n = 3$$

22. જો $3^{(x-y)} = 27$ અને $3^{(x+y)} = 243$ તો $x = \underline{\hspace{2cm}}$

$$\therefore 3^{(x-y)} = 27 \quad 3^{(x+y)} = 243$$

$$\therefore 3^{(x-y)} = 3^3 \quad 3^{(x+y)} = 3^5$$

$$\therefore x-y=3 \quad x+y=5$$

સમીકરણ-1 અને સમીકરણ-2 નો સરવાળો કરતાં

$$\begin{array}{r} x-y=3 \\ + \quad x+y=5 \\ \hline 2x=8 \\ \therefore x=4 \end{array}$$

23. જો $\left(\frac{9}{4}\right)^x \times \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$ તો $x = (?)$

$$\therefore \left(\frac{3^2}{2^2}\right)^x \times \left(\frac{2^3}{3^3}\right)^{x-1} = \frac{2}{3}$$

$$\therefore \left(\frac{2}{3}\right)^{-2x} \times \left(\frac{2}{3}\right)^{3x-3} = \left(\frac{2}{3}\right)^1$$

$$\therefore -2x+3x-3=1$$

$$\therefore x=1+3$$

$$\boxed{x=4}$$

24. જો $2^x = \sqrt[3]{32}$ તો $x = (?)$

$$2^x = (32)^{1/3}$$

$$\therefore 2^x = (2^5)^{1/3} \therefore 2^x = 2^{5/3}$$

$$\therefore \boxed{x = \frac{5}{3}}$$

25. $(1^3 + 2^3 + 3^3 + 4^3)^{1/2} = \underline{\hspace{2cm}}$

$$(1+8+27+64)^{1/2}$$

$$\therefore (100)^{1/2} \therefore \boxed{10}$$

26. $2^x \times 8^{1/5} = 2^{1/5}$ તો $x = \underline{\hspace{2cm}}$

$$\therefore 2^x \times (2^3)^{1/5} = 2^{1/5}$$

$$\therefore 2^x \times 2^{3/5} = 2^{1/5}$$

$$\therefore 2^x = 2^{1/5-3/5} = 2^{-2/5} = 2^{-2/5}$$

$$\therefore \boxed{x = -\frac{2}{5}}$$

27. $5^{(x+3)} = 25^{3x-4}$ તો $x = (?)$

$$\therefore 5^{(x+3)} = 25^{3x-4}$$

$$\therefore 5^{(x+3)} = (5^2)^{3x-4}$$

$$\therefore 5^{x+3} = 5^{6x-8} \therefore x+3=6x-8$$

$$\therefore 11=5x \therefore \boxed{x = \frac{11}{5}}$$

28. $\sqrt{2}, \sqrt[3]{3}$ અને $\sqrt[4]{4}$ પૈકી સૌથી મોટી સંખ્યા કઈ છે ?

$$\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}$$

$$\begin{aligned} &\therefore (2)^{1/2}, (3)^{1/3}, (4)^{1/4} \\ &\therefore 2^{6/12}, 3^{4/12}, 4^{3/12} \\ &\therefore (2^6)^{1/12}, (3^4)^{1/12}, (4^3)^{1/12} \\ &\therefore (64)^{1/12}, (81)^{1/12}, (64)^{1/12} \\ &\text{સૌથી મોટી સંખ્યા } \boxed{(81)^{1/12}} = \sqrt[3]{3} \end{aligned}$$

29. જો $5^{5x+5} = 1$ તો $x = \underline{\hspace{2cm}}$
 $5^{5x+5} = 1$
 $\therefore 5^{5x+5} = 5^0 \therefore 5x+5 = 0$
 $\therefore 5x = -5 \therefore x = -\frac{5}{5} \boxed{x = -1}$
30. $3^{x+3} + 7 = 250$ તો $x = \underline{\hspace{2cm}}$
 $\therefore 3^{x+3} + 7 = 250$

$$\begin{aligned} &\therefore 3^{x+3} = 250 - 7 = 243 \\ &\therefore 3^{x+3} = 3^5 \therefore x+3 = 5 \\ &\therefore x = 5 - 3 \therefore \boxed{x = 2} \end{aligned}$$

14. $(16)^{1.75}$ બરાબર કેટલા થાય ?

$$\begin{aligned} &\therefore (16)^{\frac{175}{100}} \therefore (16)^{7/4} \\ &\therefore (2^4)^{7/4} \therefore 2^7 \therefore \boxed{128} \end{aligned}$$

15. $3^{12} \times (2^x)^2 = 6^{12}$ તો $x = (?)$

$$\begin{aligned} &\therefore 3^{12} \times 2^{2x} = 3^{12} \times 2^{12} \\ &\therefore 2^{2x} = \frac{3^{12} \times 2^{12}}{3^{12}} \end{aligned}$$

$$\begin{aligned} &2^{2x} = 2^{12} \therefore 2x = 12 \\ &\boxed{x = 6} \end{aligned}$$

પ્રેક્ટીસના દાખલા

- $(64)^{\frac{2}{3}} \times (32)^{\frac{2}{5}} = ?$
 (અ) 8^2 (બ) 4^4
 (ક) 2^3 (ડ) 2^4
- $(-1)^9 + (-1)^{10} = \dots\dots\dots$
 (અ) -1 (બ) 2
 (ક) 0 (ડ) 1
- જો $x = 64$ હોય તો $\frac{1}{x^6} + \frac{-1}{x^6}$ નો સરવાળો શું થાય ?
 (અ) 1 (બ) $2\frac{1}{2}$
 (ક) 2 (ડ) 36
- $\left(8\right)^{\frac{2}{3}}$ ની કિંમત જણાવો.
 (અ) 16 (બ) 64
 (ક) 4 (ડ) $\frac{1}{4}$
- $(256)^{0.16} \times (256)^{0.09}$ નો જવાબ શું થાય ?
 (અ) 4 (બ) 16
 (ક) 0.25 (ડ) 256

6. $3^2 \times 3^4 = 3^x$ તો $x = ?$

$$\begin{aligned} &(\text{અ}) 6 \quad (\text{બ}) 8 \\ &(\text{ક}) 2 \quad (\text{ડ}) 1 \end{aligned}$$

7. $4^3 \div 2^4 \times 3 = \dots\dots\dots$

$$\begin{aligned} &(\text{અ}) 8 \quad (\text{બ}) 12 \\ &(\text{ક}) 2 \quad (\text{ડ}) 4 \end{aligned}$$

8. $(-4)^8 \div (-4)^2 \div (-4)^3$ ની કિંમત કેટલી થાય ?

$$\begin{aligned} &(\text{અ}) -64 \quad (\text{બ}) (-12) \\ &(\text{ક}) (-256) \quad (\text{ડ}) (-128) \end{aligned}$$

જવાબો

- (અ) 8^2 , 2. (ક) 0 , 3. (બ) $2\frac{1}{2}$, 4. (ક) 4 , 5. (અ) 4 ,
 6. (અ) 6 , 7. (બ) 12 , 8. (અ) -64